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Performance Analysis of Massive MIMO Downlink System with Imperfect Channel State Information

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ABSTRACT: We investigate the ergodic sum rate and required transmit power of a single-cell massive multiple-input multiple-output (MIMO) downlink system. The system considered in this paper is based on two linear beamforming schemes, that is, maximum ratio transmission (MRT) beamforming and zero-forcing (ZF) beamforming. What's more, we use minimum mean square error (MMSE) channel estimation to get imperfect channel state information (CSI). Compared with the perfect CSI case, both theoretical analysis and simulation results show that the system performance is different when the imperfect CSI is taken into account.
Keywords -massive MIMO; ergodic sum rate; required transmit power; linear beamforming schemes; MMSE channel estimation

I. Introduction

With the rapid development of multimedia technology and electronic technology, the mobile internet traffic surge, more and more sensor equipments and machine-to-machine (M2M) communications are also used in mobile communication networks. To enhance the business support ability, 5G (5th generation mobile communication technology) has become a hot topic in the mobile communication field for its high transmission rate and energy efficiency.

Massive MIMO is a promising approach for the physical layer of 5G system. This technology equips cellular base stations (BSs) with hundreds of antennas in order to achieve the aim of improving the data rate and energy efficiency[1] . Since its superior performance, massive MIMO system has been studied extensively. Reference [2] studied the performance of massive MIMO downlink system using ZF beamforming. Reference [3] provided the comparison of ZF and MRT performance with assuming that BS has full CSI. Reference[4] proposed a low-complexity hybrid precoding scheme used in massive MIMO system, the proposed scheme is proved to approach the performance of the virtually optimal yet practically infeasible full-complexity ZF precoding. The previous works have been drawn good conclusions about the performance of massive MIMO system, however, they have not taken imperfect CSI into account, and their results could be more comprehensive if they consider that full CSI is hardly available by BS in actual scene. Actually owing to the existence of pilot contamination[5] and latency Effects[6] , BSs in practical application are difficult to obtain full CSI. As a result, considering imperfect CSI has a certain reference value in perfecting the theory of massive MIMO.

In this paper, we analysis the ergodic sum rate and required transmit power of a massive MIMO downlink system using MMSE channel estimation to obtain CSI instead of assuming that the BS has perfect CSI in order to realize the effectiveness and the energy efficiency of a massive MIMO downlink system with imperfect CSI. Two common linear beamforming schemes are considered in this paper, that is, MRT and ZF. The simulation results are compared with the results in Reference[3] and [7] to illustrate the performance difference between imperfect CSI and perfect CSI.

The rest of the paper is organized as follows. The system model is introduced in Section II. Section III describes the performance analysis. Section IV gives the simulation results and conclusion of this paper is provided in Section V.

II. System Model

We consider a single cell massive MIMO downlink system, which includes one BS equipped with M antennas, and K single-antenna user equipments (UEs) that share the same bandwidth. To make sure that the linear beamforming schemes make sense,we assume that$M \gg K$.

2.1 Channel Model

Let $\mathbf{F}$ be a linear beamforming matrix, that is, $\mathbf{F} = [\mathbf{f}_1, \mathbf{f}_2, \ldots, \mathbf{f}_k, \ldots, \mathbf{f}_K]$, where $\mathbf{f}_k$ is the beamforming vector of user k . We define $\mathbf{H}$ as the channel matrix, $\mathbf{h}_k$ is the channel vector between the BS and user k. The received vector at the users is given by

$$y = \sqrt{p_d}\mathbf{HFx} + \mathbf{n} \quad (1)$$

wherep_d represents the downlink transmission power. **x** is a $K \times 1$ user data vector where is x_k a data symbol of user *k*. The noise vector **n**are independent and identically distributed (i.i.d) complex Gaussian random variables with zero mean and 1 unit variance, that is,$\mathbf{n} \sim \mathcal{CN}(0,1)$. The signal received by the user *k*after using the linear beamforming scheme is given by

$$y_k = \sqrt{p_d}\mathbf{h}_k\mathbf{f}_k\mathbf{x}_k + \sqrt{p_d}\sum_{i=1,i\neq k}^{K}\mathbf{h}_k\mathbf{f}_i\mathbf{x}_i + \boldsymbol{n} \quad (2)$$

On the right hand side of equation(2), the first part is the desired signal term, the second part is the interference term of other users, and the third part is the noise.

Using(2), the power of the desired signal can be written as $E\{z_k z_k^H\} = p_d|\mathbf{h}_k\mathbf{f}_k|^2$. Assuming that inter-user are independent, the power of the interference can be written as$E\{I_k I_k^H\} = p_d\sum_{i=1,i\neq k}^{K}|\mathbf{h}_k\mathbf{f}_i|^2$.Then the signal to interference plus noise ratio *(SINR)* of user *k* can be expressed as[7]

$$SINR_k = \frac{p_d|\mathbf{h}_k\mathbf{f}_k|^2}{p_d\sum_{i=1,i\neq k}^{K}|\mathbf{h}_k\mathbf{f}_i|^2+1} \quad (3)$$

2.2 Channel Estimation

In massive MIMO system, uplink and downlink channels tend to be reciprocal, and the BS can exploit the channel reciprocity to obtain full CSI theoretically. However, in practice, full CSI may not be directly available due to pilot contamination and feedback delay.

In order to get finite CSI, we use MMSE channel estimation. The channel matrix can be estimated as

$$\widehat{\mathbf{H}} = \xi\mathbf{H} + \sqrt{1-\xi^2}\mathbf{E} \quad (4)$$

whereξ is the reliability of the estimation,and $\mathbf{E} \sim \mathcal{CN}(0,1)$represents the error matrix.

2.3Linear Beamforming Schemes

We consider two classical linear beamforming schemes in this work,which include MRT and ZF.

When perfect CSI is available, the MRT beamformer is given as $\mathbf{F} = \mathbf{H}^H$ [8] and the ZF beamformer is given as $\mathbf{F} = \mathbf{H}^H(\mathbf{HH}^H)^{-1}$[8].In practical application,the beamformer use the estimated channel so that the MRT beamformer is given as

$$\mathbf{F} = \widehat{\mathbf{H}}^H \quad (5)$$

and the ZF beamformer is given as

$$\mathbf{F} = \widehat{\mathbf{H}}^H(\widehat{\mathbf{H}}\widehat{\mathbf{H}}^H)^{-1} \quad (6)$$

where$\widehat{\mathbf{H}}^H$represents the complex conjugate transpose of the estimated channel matrix.

III. Performance Analysis

In (5) and (6), it was shown that channel estimation has great influence on linear bemforming schemes. This means that imperfect CSI has a great effect on performance of massive MIMO downlink system. In this sction, we derive the expression for the system performance, which includes ergodic sum rate and required downlink transmit power.

3.1 Ergodic Sum Rate

Ergodic sum rate can be used to describe the effectiveness of a massive MIMO system. For simply analyzing the ergodic sum rate of the system, we assume that the total downlink power is fixed and equally shared among all the users in a single cell.

Using (3), the rate of user *k* can be expressed as

$$R_k = \log_2(1 + SINR_k) \quad (7)$$

Then for K number of users, the ergodic sum rate is given as

$$R_{sum} = \sum_{k=1}^{K} E\{R_k\}(8)$$

For the derivations we denote

$$\alpha = \frac{M}{K} \quad (9)$$

Combining(3)~(9), the ergodic sum rate with MRT and ZF can be respectively expressed as

$$R_{sum}^{mrt} = K \cdot \log_2(1 + \frac{\xi^2 p_d \alpha}{p_d+1}) \quad (10)$$

$$R_{sum}^{zf} = K \cdot \log_2[1 + \frac{\xi^2 p_d(\alpha-1)}{(1-\xi^2)p_d+1}] \quad (11)$$

Set (10) equal to (11), then get

$$\alpha = \frac{p_d+1}{\xi^2 \cdot p_d} \quad (12)$$

Equation(12) shows that a cross point is existing in MRT and ZF curves, it means that neither MRT nor ZF achieves higher ergodic sum rate in the full range of α.

3.2 Required downlink transmit power

A communication system is more energy efficient when less transmit power is needed to reach a targeted rate with satisfactory quality of service (QoS). For simply analyzing the required downlink transmit power of the massive MIMO system, we assumed that the total downlink transmit power and the ergodic sum rate is equally divided among all the users.

Changing the form of equation (10), we obtain

$$p_d^{mrt} = \frac{e^{\frac{\ln 2 \cdot R_{sum}^{mrt}}{K}} - 1}{(\xi^2 \cdot \alpha) - (e^{\frac{\ln 2 \cdot R_{sum}^{mrt}}{K}} - 1)} \tag{13}$$

Substituting (9) into (13) gives

$$p_d^{mrt} = \frac{K \cdot (e^{\frac{\ln 2 \cdot R_{sum}^{mrt}}{K}} - 1)}{(\xi^2 \cdot M) - K \cdot (e^{\frac{\ln 2 \cdot R_{sum}^{mrt}}{K}} - 1)} \tag{14}$$

which is the required downlink transmit power with MRT beamforming scheme.

Similarly, changing the form of equation (11), we obtain

$$p_d^{zf} = \frac{e^{\frac{\ln 2 \cdot R_{sum}^{zf}}{K}} - 1}{[(\xi^2 \cdot (\alpha - 1)] - [(\xi^2 - 1) \cdot \left(e^{\frac{\ln 2 \cdot R_{sum}^{zf}}{K}} - 1\right)]} \tag{15}$$

Substituting (9) into (15) gives

$$p_d^{zf} = \frac{K \cdot (e^{\frac{\ln 2 \cdot R_{sum}^{zf}}{K}} - 1)}{[(\xi^2 \cdot (M - K)] - [K \cdot (\xi^2 - 1) \cdot \left(e^{\frac{\ln 2 \cdot R_{sum}^{zf}}{K}} - 1\right)]} \tag{16}$$

which is the required downlink transmit power with ZF beamforming scheme.

Comparing (14) with (16), it shows that imperfect CSI has a greater effect on ZF.

IV. Simulation Results

In this section, we conduct the simulation using MATLAB software to validate the theoretical analysis in section III.

Figure 1 depicts the ergodic sum rate versus the number of BS antennas in both perfect CSI and imperfect CSI cases with the number of mobile users fixed to 5, the ξ^2 fixed to 0.5 and the BS downlink transmit power equal to 0*dB*. The results shows that as the number of BS antennas increase, the ergodic sum rate for MRT and ZF increases in both perfect CSI and imperfect CSI cases, in addition, perfect CSI case achieves higher ergodic sum rate than imperfect CSI case. On the other hand,an obvious cross point is existing in imperfect CSI curves. When the value of *M* is smaller than the abscissa of the cross point, MRT achieves higher ergodic sum rate and ZF achieves higher ergodic sum rate when the value of *M* is larger than the abscissa of the cross point.

Figure 2 demonstrates the ergodic sum rate versus the number of BS antennas in both perfect CSI and imperfect CSI cases with the number of BS antennas fixed to 100, the ξ^2 fixed to 0.5 and the BS downlink transmit power equal to 0*dB*. The results shows that perfect CSI case achieves higher ergodic sum rate than imperfect CSI case for both MRT and ZF. Besides, as the number of users increase,the ergodic sum rate for MRT increases, however, the ergodic sum rate for ZF increases first, then decreases. It means that there is an optimal number of users for the highest ergodic sum rate when ZF is used. Moreover,imperfect CSI makes the optimal number of users and its corresponding ergodic sum rate smaller.

Figure 3, 4 and 5 depict the required downlink transmit power versus the number of BS antennas in both perfect CSI and imperfect CSI cases with the number of mobile users fixed to 10, the ξ^2 fixed to 0.5 and the targeted ergodic sum rate is respectively equal to 5*bits/s/Hz*, 10*bits/s/Hz* and 15*bits/s/Hz*. The results in three figures show that as the number of BS antennas increases,the required downlink transmit power for MRT and ZF decreases in both perfect CSI and imperfect CSI cases. Furthermore, with increasing the targeted ergodic sum rate, two MRT curves, which include the perfect CSI one and the imperfect CSI one, become closer with respect to the ZF curves. It means that the difference of MRT between perfect CSI and imperfect CSI cases is becoming smaller compared to the ZF one when the ergodic sum rate achieves higher. In other words, the imperfect CSI has a greater effect of required downlink transmit power on ZF and the effect is more notable when the targeted ergodic sum rate is higher.

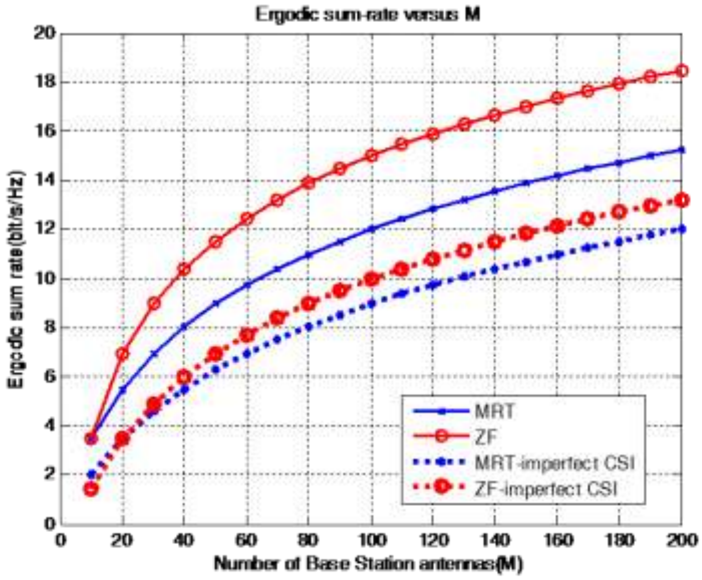

Fig.1Ergodic sum rate versus the number oftransmit antennas with perfect CSI and imperfect CSI at K=5, ξ^2=0.5, p_d=0dB

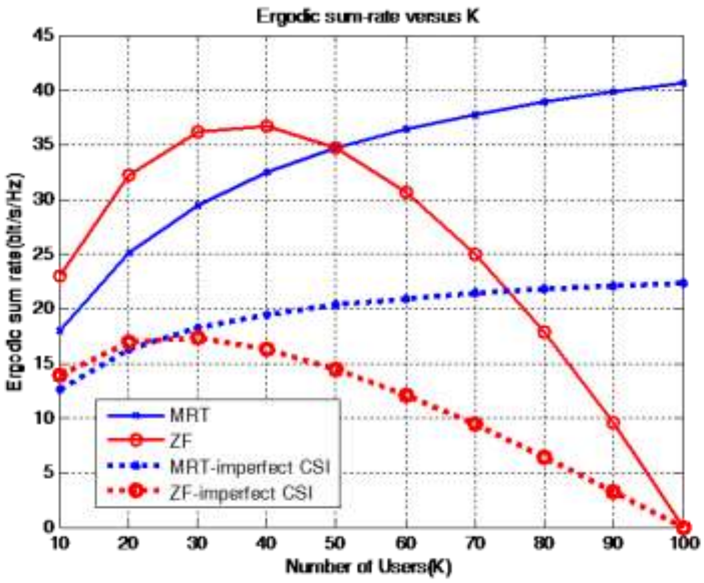

Fig.2Ergodic sum rate versus the number of users with perfect CSI and imperfect CSI at M=100, ξ^2=0.5, p_d=0dB

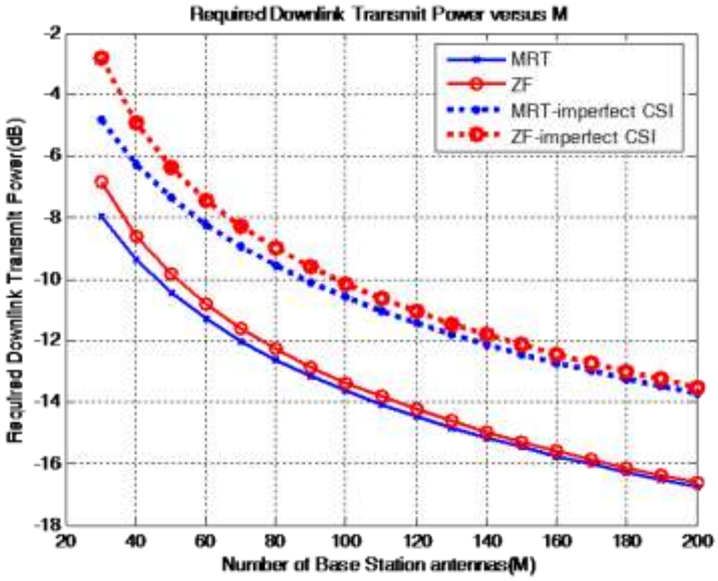

Fig.3 Required downlink transmit power versus the number of transmit antennas with perfect CSI and imperfect CSI at K=10,ξ^2=0.5,$R_{sum}^{mrt} = R_{sum}^{zf} = 5 bits/s/Hz$

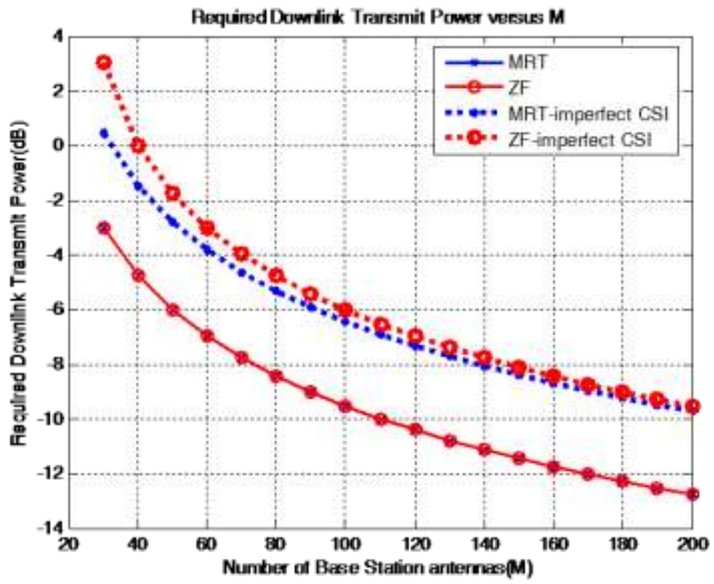

Fig.4Required downlink transmit power versus the number of transmit antennas with perfect CSI and imperfect CSI at K=10, ξ^2=0.5,$R_{sum}^{mrt} = R_{sum}^{zf} = 10 bits/s/Hz$

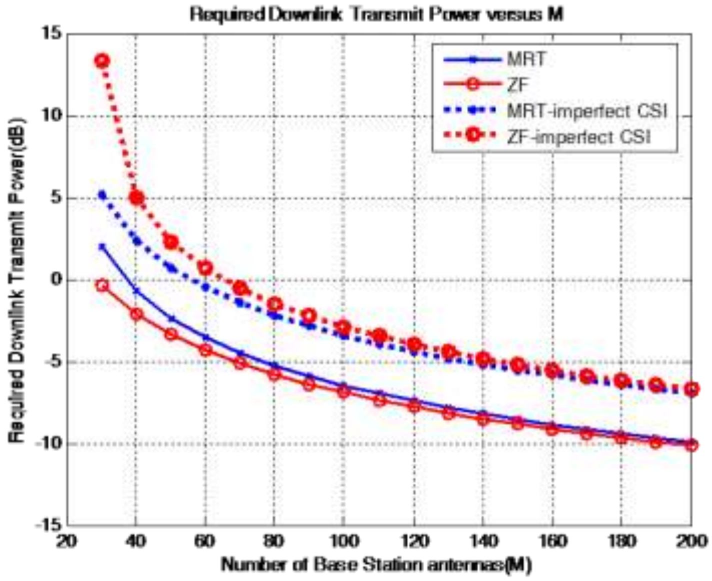

Fig.5Required downlink transmit power versus the number of transmit antennas with perfect CSI and imperfect CSI at K=10, ξ^2=0.5,$R_{sum}^{mrt} = R_{sum}^{zf} = 15bits/s/Hz$

V. Conclusion

In this paper, we have analysed the massive MIMO downlink system performance with imperfect CSI, which included ergodic sum rate and required downlink transmit power. MRT and ZF beamforming schemes were considered and MMSE channel estimation is used to obtain CSI. We found that neither MRT nor ZF achieves higher ergodic sum rate in all cases.Besides,against the required downlink transmit power,imperfect CSI has a greater effect on ZF than MRT when the target ergodic sum rate is fixed. This work have certain guiding significance for the actual design of massive MIMO system.

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